

# Maxent in Experimental $2 \times 2$ Population Games

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## Abstract

In mixed strategy  $2 \times 2$  population games, the realization of maximum entropy (Maxent) is of the theoretical expectation. We evaluate this theoretical prediction in the experimental economics game data. The data includes 12 treatments and 108 experimental sessions in which the random match human subjects pairs make simultaneous strategy moves repeated 200 rounds. Main results are (1) We confirm that experimental entropy value fit the prediction from Maxent well; and (2) In small proportion samples, distributions are deviated from Maxent expectations; interesting is that, the deviated patterns are significant more concentrated. These experimental finding could enhance the understanding of social game behavior with the natural science rule — Maxent.

*Keywords:* experimental economics, Maxent, mixed strategy Nash equilibrium, population game, 2 by 2 game

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## 1. Introduction

The principle of Maxent is introduced by Jaynes [1]. By capture the symmetry in stochastic behaviors, Maxent methodology can provide rich information basing on very limit information [2, 3]. Maxent has gained its wide applications in natural science and engineering. In social science, Maxent approach has also gained its applications, e.g., to prove the existence of market equilibrium [4, 5], to investigate wealth and income distribution [6, 7], to explore firm growth rates [8] and to build fundamental behavior model [3]. Theoretical interpreting or modeling of the distributions of social outcomes with Maxent is growing.

Considering the importance of Maxent in social interaction systems, experimental investigation this fundamental rule is necessary [9]. Only quite recently, entropy is firstly measured out in experiments to evaluate social outcomes by Bednar et.al. [10] and Cason et.al. [11]; Then, Xu. et.al [12] find that fixed-paired two-person constant-sum  $2 \times 2$  games obey Maxent [12]. These are the only experimental works, to the best of our knowledge, relates to entropy or Maxent till now.

Comparing with fixed-paired games, population games are more general models for social behavior [13]. Investigating Maxent in population game is expected because: (1) Maxent lacks of least experimental population games supporting; (2) The experimental precision of Maxent in fundamental experiments is unclear. Robustness experimental results could be the references for future investigations relates to Maxent — a hub in interdisciplinary.

The main aim of this paper is testing Maxent in experimental population  $2 \times 2$  games. Randomizing and independence in mixed strategy systems is game theoretician expectation [14], meanwhile, randomizing and independence will lead to the realization of Maxent [15]. In an ideal (explanations see Section 2.1) mixed strategy experimental data set from ref. [16], we find that (1) experimental entropy values fit the predictions from Maxent well; and (2) whenever the experimental distributions are deviated from Maxent predictions, the deviated distribution patterns are concentrated significant. Quantitatively, this report provides the precision of Maxent in experimental economics systems.

This report is organized as following. In next section the fundamental  $2$  by  $2$  games experiments are reviewed, meanwhile, the reasons for the choosing of this data set are given. In *3rd* session, the experimentrics for entropy and distribution, calculation for Maxent expectations and approximate criterion

are explained. Results are reported in 4th section. Then, the experimetrics and the implication of our finding on Maxent are discussed briefly and conclusion last.

## 2. Experiments, Data and Presentation

### 2.1. Experiments and Data

Mixed strategy  $2 \times 2$  game is one of the most fundamental and the simplest game used widely from student textbooks to frontier research in interdisciplinary. To investigate the performance of Maxent in human subjects social interaction systems, using this kind of experimental game data to is natural. The data we use in this report comes from Selten and Chmura's experiments [16].

The experiments contain 12 games, 6 constant sum games, and 6 nonconstant sum games. Games were run with 12 independent subject groups for each constant sum game and 6 independent subject groups for each nonconstant sum game. Each independent subject group consisted of four players  $X$  and four players  $Y$ , interacting anonymously over 200 periods with random matching. There were 864 student subjects of the University of Bonn altogether used in these experiments. All of the game are of unique mixed strategy Nash equilibrium systems. In a session with 200 round, the distribution is suggested to be stationary. In experimetrics view, in these experiments, the treatment sample number is 12 at treatment level, and the group sample number is 108 at session (group) level. At each experimental round, the 8 individual records can be combined as a social outcome of this round; So in a session, there are 200 observations in strategy space in which the distribution can be estimated.

Table 1: Payoff Matrix of  $2 \times 2$  Games

	$Y_1$	$Y_2$
$X_1$	$a_{11}, b_{11}$	$a_{12}, b_{12}$
$X_2$	$a_{21}, b_{21}$	$a_{22}, b_{22}$

The reasons to choose this data set are (1) The games being mixed strategy, so randomization and independence social outcomes can be expected. (2) The unique equilibrium, so each sample has its own observation mean; (3) The variety of the experimental parameters, so the results less bias; (4)

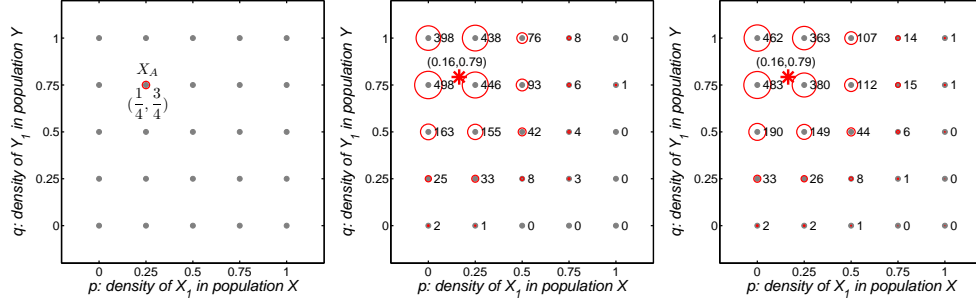


Figure 1: Presentation of the social states lattices and distribution. (Left) A social state which means one in the four agents in  $X$ -population uses  $X_1$ , meanwhile three out of the four agents in  $Y$  use  $Y_1$ . The degree of degenerate is 16 (see the interpretation for Eq. 4). (Middle) Experimental distribution, data from the Game-3 with 2400 observations, red star is the mean observation ( $\bar{O}$ ) [16]. (Right) Theoretical distribution from Maxent with the red star ( $\bar{O}$ ) servicing as the given constraints for the Game-3 [16]. Numerical labels are the observations.

67 Sufficient samples for there are 108 groups samples, so the confidence of the  
68 results could be expected; (5) The distribution of mixed strategy Nash equi-  
69 librium in strategy space is uniform, so the samples from the data could also  
70 be less bias for evaluating social performance. These characters make this  
71 data set to be ideal and the unique for Maxent investigation.

72 As subjects (Players  $X$  or players  $Y$ ) do not change their roles in a game,  
73 subjects in each role can be modeled as a population. So, these games can be  
74 regarded as two-population games. Supposing for the first population  $X$ , the  
75 strategy set is  $\{X_1, X_2\}$  for each agent; similarly, in the second population  $Y$ ,  
76  $\{Y_1, Y_2\}$ . The payoff matrix for the 12 games can be presented mathematically  
77 in Table 1 in which, in each cell, the numeric in left (right) is the payoff for  
78 the agent from the player from  $X$  ( $Y$ ). For the 12 treatments, the payoff  
79 matrix cells are shown in column 2-5 in Table 2 referring to Table 1.

## 80 2.2. Presentation of Social State

81 A two-population game's outcomes can be presented in a unit square  
82 strategy space [17, 18] or in a discrete state lattice [19, 20]. If there are 4  
83 agents in each of the two population ( $N=4$ ), an observed instantaneous state  
84 should be  $x := (\frac{i}{N}, \frac{j}{N}) \in \mathbb{X}$ , herein  $\mathbb{X}$  is the populations strategy state space  
85 and  $\mathbb{X} = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\} \otimes \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ , and  $\frac{i}{N}$  ( $\frac{j}{N}$ ) is the density of  $X_1$  ( $Y_1$ ) in  
86  $X$  ( $Y$ ). Figure 2.2 (left) is an illustration of the space and the states. The

unit square  $5 \times 5$  lattice (gray dots) is the space  $\mathbb{X}$ . Dots are states. For example, the dot  $x_A = (\frac{1}{4}, \frac{3}{4})$  is a state.

At each round, one observation is gained. So in data, for each treatment 1-6 (7-12) there are 2400 (1200) observations; For each group there are 200 observations. All observations distribute in the 25 dots lattice.

Motivation of using social state presentation is in two aspects. First, instead of concern the diversity of individual behavior, we concern the performance of social outcomes more [17, 18, 21, 13]. And second, in this presentation, significant more observations can be harvested from experiment so the regular of social behavior can be evaluated. In sequential strategy presentation, there are  $2^8$  states in above experiments; meanwhile, in social state presentation, it is 25 states.

### 3. Measurement and Criterion

#### 3.1. Measurement

Experimental distribution ( $O_{ij}$ ) at state  $(\frac{i}{N}, \frac{j}{N})$  can be obtained in data. Every experimental round, an observation is gained in one state in the  $5 \times 5$  lattice. The distribution  $O_{ij}$  can be obtained by pooling up all observations from the experimental rounds. For example, in Fig. 2.2(middle), state- $(\frac{1}{4}, \frac{3}{4})$  is labeled as 466 means  $O_{(\frac{1}{4}, \frac{3}{4})} = 466$ . In normalized form, density of state  $\rho_{ij}$  can be calculated as  $\rho_{ij} := O_{ij}/T$  in which  $T$  is the total experimental rounds.

The entropy ( $S$ ) can be calculated with a given distribution. Recall that, the original metric for entropy is

$$S = - \sum_k \rho(k) \log_\gamma \rho(k), \quad (1)$$

in which  $k$  is an element in the individual sequential states set whose independent element number is  $2^8$ ; and  $\gamma$  is the potential information in one fundamental observation [22]. This point differs from ref. [10, 11] and should be discussed last paragraph in Section 5.1. In social state presentation, the entropy can be expressed as

$$S = - \sum_{ij} [\rho_{ij} \log_\gamma \rho_{ij} - \rho_{ij} \log_\gamma D_{ij}]; \quad (2)$$

here,  $D_{ij}$  is the degeneracy of state  $(\frac{i}{N}, \frac{j}{N})$  and its arthogram sees the interpreting of Eq 4.

117 Experimental mean observation ( $\bar{O}$ ) can be calculated from ( $O_{ij}$ ) as

$$\bar{O} = \sum_{ij} \rho_{ij} \left( \frac{i}{N} \hat{e}_p + \frac{j}{N} \hat{e}_q \right) \quad (3)$$

118 in which  $\rho_{ij}$  is the density of state at  $(\frac{i}{N}, \frac{j}{N})$  and  $(\hat{e}_p, \hat{e}_q)$  are the two unit  
 119 vectors.  $\bar{O}$  is a 2-dimensional vector and could be presented as  $(\bar{O}_p, \bar{O}_q)$  who  
 120 server as constraint conditions, which is most often used [23], for Maxent  
 121 estimation.

### 122 3.2. Theoretical expectation

123 Maxent *requires* that the noise process be assigned a *binomial distribution*  
 124 in discrete case, independent and identically distributed — this describes our  
 125 state of knowledge about the noise [15]. The theoretical expected distribution  
 126 ( $E_{ij}$ ) at the state  $(\frac{i}{N}, \frac{j}{N})$ , with the experimental mean observation ( $\bar{O}$ ), can  
 127 be calculated as [15]

$$E_{ij} = C_N^i C_N^j \bar{O}_p^i (1 - \bar{O}_p)^{N-i} \bar{O}_q^j (1 - \bar{O}_q)^{N-j}, \quad (4)$$

128 in which  $N$  is the population size ( $N=4$  in our case),  $C_N^k$  is binomial coef-  
 129 ficient and  $C_N^i C_N^j$  is the degree of degenerate (denoted as  $D_{ij}$  above) of the  
 130 state; For example, at  $x_A$  in Fig. 2.2(left),  $D_{(\frac{1}{4}, \frac{3}{4})}$  is  $C_4^1 C_4^3 = 16$ . Now, with  
 131 Eq. 2 and Eq. 4, theoretical expected entropy ( $S_t$ ) can be obtained.

### 132 3.3. Approximate estimators of the goodness of fit of Maxent

133 On entropy view, the approximate criterion to test the goodness of Jaynes  
 134 Maxent prediction is the entropy concentration theorem (ECT). The theorem  
 135 states that out of all distributions that satisfy the observed data (moments), a  
 136 significantly large portion of these distributions are concentrated sufficiently  
 137 close to the one of maximum entropy [24, 2]. The measured entropy from  
 138 experiment  $S_e$  can be expressed as [24]

$$S_t - \Delta S \leq S_e \leq S_t, \quad (5)$$

139 in which  $S_t$  is evaluated from Maxent; Meanwhile,  $\Delta S$  can be calculated by

$$2M\Delta S = \chi_k^2(1 - F), \quad (6)$$

140 in which  $(1 - F)$  is the upper tail area (denoted as  $p := 1 - F$  called as  
 141 statistical significant index);  $k = n - 2 - 1$  is the degrees of freedom in

142 which the 2 is the constrained freedoms due to  $\bar{O}$  (the experimental mean  
 143 observation),  $n=25$  is the number of the social states, and then the freedom  
 144 in the experiments are of 22. From  $\chi^2$  table,  $\chi^2_{22}(1 - 0.95)$  is 33.92. So  
 145 the  $\Delta S$  values are of 0.0071, 0.0141 and 0.0848 for the Treatment 1-6 game  
 146 ( $M=2400$ ) conditions, the Treatment 7-12 game ( $M=1200$ ) conditions and  
 147 the 108 group ( $M=200$ ) condition, respectively.

148 On distribution view, the  $\chi^2$  goodness of fit test is used to summarize the  
 149 discrepancy; The  $\chi^2$  statistic is

$$\chi^2 = \sum_{i \in S} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}; \quad (7)$$

150 Here  $O_{ij}$  ( $E_{ij}$ ) is the experimental (theoretical) distribution and is given by  
 151 Eq. 2 (Eq. 4).

152 One point needs to emphasis. Theoretically, both of the criterion above  
 153 for Maxent are indirectly statistical methods, or saying, both of the crite-  
 154 rion have its shortcoming facing experimental economics condition. In the  
 155 theoretical derivation of  $\Delta S$ , the deviate rate of observe from theoretical  
 156 expected rate needs to be sufficient small and so high order term can be ig-  
 157 nored. However, in the edges of the 25 social states lattice, the experimental  
 158 observation is rare. Practically, the requirement of the sufficient small can  
 159 not be satisfied. On the other side, the  $\chi^2$  goodness of fit test is not valid  
 160 if the expected frequencies are too small. There is no general agreement on  
 161 the minimum expected frequency allowed, even though values of 5 are often  
 162 used. However, in the edge of the strategy lattice, to gain 5 observations is  
 163 not easy especially for the mean observation closes to the edge of the lattice.  
 164 It is a dilemma, if the few observed state are pooled, the good performance of  
 165 the theoretical prediction on these states is hidden, the degree of freedom is  
 166 lower. So we call above two criterions as *approximate estimators* for Maxent  
 167 in these experimental conditions.

## 168 4. Results

169 To test the fit within the Maxent expectations and the experimental re-  
 170 sults, the two observable (1) entropy and (2) distribution are used. Results  
 171 and the supporting materials are reported following.



172 *4.1. Experimental Entropy and Precision of Maxent prediction*

173 Experimental entropy values can be calculated from the experimental  
174 distribution (Eq. 2). Meanwhile, the theoretical entropy can be calculated  
175 using the mean observation ( $\bar{O}$ ) and Eq. 2 with Eq. 4.

176 Figural results for eye guide, shown in Figure 4.1, demonstrates the pre-  
177 cision of the Maxent prediction meet the experimental entropy values in  
178 treatment level (12 samples, left) and group level (108 samples, right).

179 Numerical results of the accuracy for the 108 sessions is shown in Table 3;  
180 The proportion of the deviation  $D_{t-e}$  ( $D_{t-e} := 1 - \frac{S_e}{S_t}$ ) are listed along the  
181 treatment and group (in T-G columns). Directly statistical results for the  
182 deviation entropy ( $D_{t-e}$ ) are group by treatment is shown in Table 2. So,  
183 we comes to the conclusion that, prediction of the Maxent suggestion is  
184 supported in experimental economics data at accuracy about  $0.020 \pm 0.002$   
185 level (see the last raw in Table 2). In words, in view of entropy values in the  
186 samples, the experimental entropy meets the Maxent prediction well.

Table 2: Payoff matrix and the Deviation of  $S_e$  from  $S_t$  of 12 Treatments

T-Gs	$a_{11}, b_{11}$	$a_{12}, b_{12}$	$a_{21}, b_{21}$	$a_{22}, b_{22}$	Mean	S.E.	[99% c.i.]
1, 12	10, 8	0, 18	9, 9	10, 8	0.030	0.009	0.008, 0.052
2, 12	9, 4	0, 13	6, 7	8, 5	0.017	0.003	0.011, 0.024
3, 12	8, 6	0, 14	7, 7	10, 4	0.024	0.006	0.009, 0.038
4, 12	7, 4	0, 11	5, 6	9, 2	0.011	0.001	0.009, 0.014
5, 12	7, 2	0, 9	4, 5	8, 1	0.017	0.005	0.005, 0.029
6, 12	7, 1	1, 7	3, 5	8, 0	0.015	0.003	0.008, 0.022
7, 6	10, 12	4, 22	9, 9	14, 8	0.037	0.013	0.003, 0.071
8, 6	9, 7	3, 16	6, 7	11, 5	0.016	0.002	0.011, 0.020
9, 6	8, 9	3, 17	7, 7	13, 4	0.013	0.003	0.006, 0.021
10, 6	7, 6	2, 13	5, 6	11, 2	0.023	0.006	0.008, 0.038
11, 6	7, 4	2, 11	4, 5	10, 1	0.013	0.002	0.009, 0.017
12, 6	7, 3	3, 9	3, 5	10, 0	0.019	0.007	0.002, 0.037
Total					0.020	0.002	0.015, 0.023

187 A theoretical approximate estimation of the low-bound of experimental  
188 entropy ( $S_e$ ) is in Eq. 6. Using  $\Delta S$  ( $:= S_t - S_e$ ) from these 108 experimen-  
189 tal sessions as samples, the mean deviation ( $\Delta S$ ) from Maxent is 0.0158,  
190 the standard error is 0.0012 and the 99% confident interval (c.i.) is be-  
191 tween [0.0125 0.0190]. Calculated from Jaynes ECT (Eq. 6), the criterion

192 of up bound of  $\triangle S = 0.0848$  which is large than up bound of the 99% c.i.  
193 significant. This approximate estimation supports above result that the ex-  
194 perimental entropy meets the Maxent prediction well.

Table 3: Deviation ( $D_{t-e} := 1 - S_e/S_t$ ) of Entropy at Group Level

T-G	$D_{t-e}$	T-G	$D_{t-e}$	T-G	$D_{t-e}$	T-G	$D_{t-e}$	T-G	$D_{t-e}$	T-G	$D_{t-e}$
1-1	.042	2-7	.03	4-1	.01	5-7	.066	7-1	.098	10-1	.025
1-2	.062	2-8	.006	4-2	.012	5-8	.013	7-2	.013	10-2	.018
1-3	.012	2-9	.009	4-3	.012	5-9	.022	7-3	.016	10-3	.05
1-4	.009	2-10	.012	4-4	.01	5-10	.017	7-4	.022	10-4	.018
1-5	.024	2-11	.022	4-5	.009	5-11	.009	7-5	.032	10-5	.018
1-6	.007	2-12	.017	4-6	.022	5-12	.010	7-6	.04	10-6	.01
1-7	.012	3-1	.033	4-7	.009	6-1	.009	8-1	.017	11-1	.018
1-8	.023	3-2	.015	4-8	.013	6-2	.009	8-2	.013	11-2	.014
1-9	.015	3-3	.025	4-9	.011	6-3	.016	8-3	.016	11-3	.010
1-10	.110	3-4	.007	4-10	.008	6-4	.014	8-4	.009	11-4	.009
1-11	.016	3-5	.008	4-11	.011	6-5	.010	8-5	.021	11-5	.017
1-12	.027	3-6	.012	4-12	.01	6-6	.010	8-6	.019	11-6	.012
2-1	.016	3-7	.006	5-1	.011	6-7	.012	9-1	.01	12-1	.015
2-2	.016	3-8	.007	5-2	.013	6-8	.022	9-2	.017	12-2	.013
2-3	.005	3-9	.014	5-3	.01	6-9	.011	9-3	.004	12-3	.014
2-4	.031	3-10	.056	5-4	.013	6-10	.041	9-4	.011	12-4	.011
2-5	.016	3-11	.043	5-5	.01	6-11	.015	9-5	.023	12-5	.011
2-6	.029	3-12	.059	5-6	.013	6-12	.012	9-6	.016	12-6	.053

#### 195 4.2. Distribution and Deviation Pattern

196 Distribution ( $O_{ij}$ ) and the mean observation  $\bar{O}$  in an experimental session  
197 data can be obtained (Eq. 3). In the unit square of strategy lattices, for  
198 example, the size of the yellow cycles in figure 4.2 indicates the experimental  
199 distribution ( $O_{ij}$ ) and the red star indicates the  $\bar{O}$  vector for a session; using  
200 this  $\bar{O}$ , theoretical distribution ( $E_{ij}$ ) for a session can be evaluated (Eq. 4)  
201 directly.

202 An indirect method is used to evaluate the deviation of experimental  
203 distribution from Maxent predictions. Using Eq.7 the  $\chi^2$  for each group can

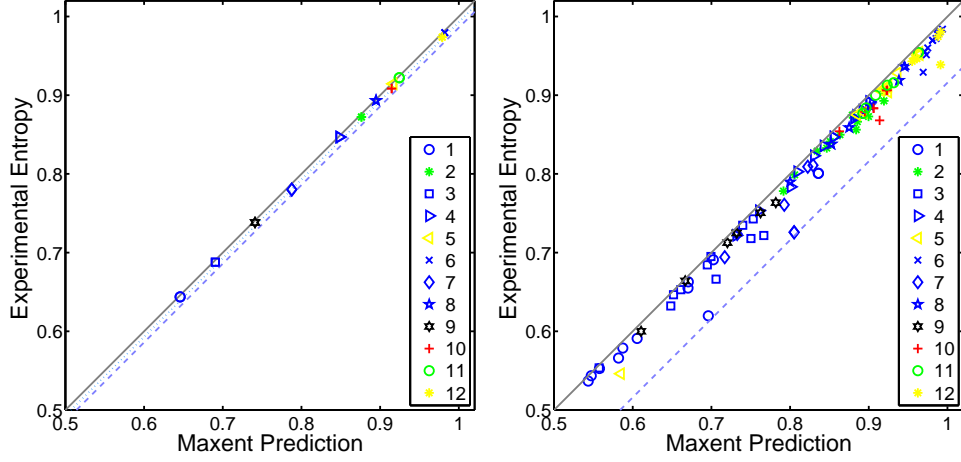


Figure 2: Experimental entropy (in vertical) vs Maxent theoretical expectation of the entropy (in horizon) in treatment level (left) and in group level (right). Legend indicate the 12 treatments (from Game 1 to Game 12 referring to ref [16] respectively). The thin dash indicates the theoretical low-bound of  $S_e$  calculated by Eq. 6.

204 be obtained. We use  $\chi_{22}^2(1 - 0.95) = 33.92$  as approximation criterion, in total  
 205 108 group, there are 27 groups whose  $\chi^2$  is larger than 33.92.

206 Figure 4.2 presents the distribution and deviation of the whole 27 groups.  
 207 Each of the sub-plot presents a given T-G (treatment-group) group. Results  
 208 of the distribution ( $O_{ij}$ ) are in yellow; Meanwhile, the positive deviation  
 209 ( $O_{ij} - E_{ij} > 0$ ) is plotted in red, alternatively, the negative deviation is ( $O_{ij} -$   
 210  $E_{ij} < 0$ ) in blue. And the number in brackets ( $Z$ ) is the results of the sum of  
 211 distance of the deviation of distribution in which  $Z := \sum_{ij} |x_{ij} - \bar{O}|(E_{ij} - O_{ij})$ .  
 212 Statistically,  $Z$  smaller than 0 ( $p < 0.001$ ,  $t$ -test and  $H0: Z=0$ ). This means  
 213 that, when the Maxent prediction is deviation, on the contrary to more tire-  
 214 like shape, the binomial bell is, in statistical significant, more sharper.

## 215 5. Discussion and Conclusion

216 Maxent has being developed since 1970s to interpret or model the social  
 217 outcome distributions [4, 5, 6, 7, 8, 3, 2]. This report provides, firstly, the lab-  
 218 oratory population  $2 \times 2$  games supporting Maxent (shown in Fig. 4.1). The  
 219 precision of Maxent prediction on entropy values in the fundamental exper-  
 220 imental games comes up to 2%. When experimental distribution is slightly  
 221 deviates from Maxent prediction, the residual pattern is concentrated signifi-

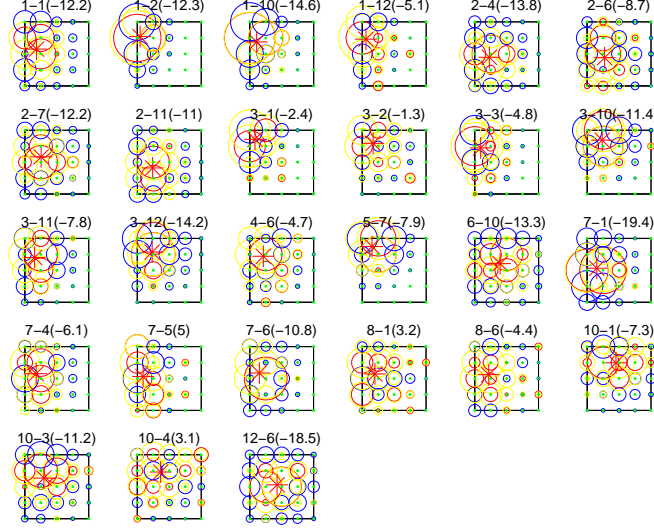


Figure 3: Pattern of the deviation ( $O_{ij} - E_{ij}$ ). Size of the red (blue) cycles indicates the positive (negative) deviation (enlarged 5 times for visible). Size of the yellow cycles indicates the experimental observation  $O_{ij}$ . In titles, on the three index X-Y(Z), X is Game.ID., Y is Group.ID, And (Z), the number in brackets, is results of the sum of deviation.

222 cant (shown in Fig. 4.2). Before interpreting the theoretical and experimental  
 223 implication, we discuss the experimetric used above.

### 224 5.1. Experimetric Technologies

225 In social state presentation, the sequential states, whose social propor-  
 226 tions are same, is pooled to one social states. This presentation has been  
 227 widely used for investigate social evolution [17, 18, 21, 13, 19, 20]. Our in-  
 228 vestigations are base on this presentation. On the other side, our results  
 229 have to be strictly constricted on social outcomes, instead of saying that the  
 230 individual behavior in the games satisfied Maxent.

231 Two approximate technologies is used in this report. First is the using of  
 232 the Jaynes concentration theorem. Indeed we use the  $\Delta S$  as a approximation  
 233 (the dash lines in Figure 4.1 as a eye guide), but the results of Maxent  
 234 well fitted do not relay on the theorem (as explained in last paragraph in  
 235 Section 3). The results of Maxent well fitted mainly based on the directly  
 236 measurement results shown in Table 2 and Table 3. Second is the using of  
 237  $\chi^2$  goodness of fit approximation to filter out more bias samples. This filter

238 out method mainly base on the  $\chi^2$  value and larger  $\chi^2$  means more deviate  
 239 from binomial. So, this method do not effect the existence of the sharper-  
 240 than-binomial pattern in the groups. So, our main results mainly base on  
 241 the directly measurements (Table 2, Table 3 and Fig. 3) instead of relaying  
 242 on the two approximate statistical technologies.

243 On experimetrics for entropy in Eq. 2, the root ( $\gamma$ ) is not 2 as ref. [10,  
 244 11, 12]. We strictly fellow ref. [22] in which the root is the number of all  
 245 microstate in a device. We set  $\gamma=2^8$  because for each of the 8 subjects has  
 246 its own two options, the information in this *social device* is 8-bit. In this  
 247 way,  $S$  has it natural interval  $[0, 1]$ , and more important, different games can  
 248 have comparable interval.

## 249 5.2. Implication on Theory

250 In mixed strategy game, randomizing using each pure strategy in a certain  
 251 proportion and playing independent — these are the fundamental prediction  
 252 of game theory [14]. Meanwhile, in randomization and independence behav-  
 253 iors system, Maxent is a promising [15] or as a prior [3]. Using human subjects  
 254 data, this report makes these two fundamental predictions meet. The finding  
 255 demonstrates that social behavior could have *common background* as natural  
 256 science.

257 Maxent could help for game theory. Mixed strategy Nash equilibrium is  
 258 remarkably supported in experiments [21, 18]. However, the mixing is still  
 259 problem (e.g., Ch.6 [25]). Proposal are are developed — like purification [26],  
 260 population (people play the same game with different players over time and  
 261 a mixed strategy is just a distribution of different pure strategies in popula-  
 262 tion), and randomization device (players are playing a pure strategy based  
 263 on some randomization device), and so on. Facing these proposal, a naive  
 264 directly suggestion is, at least in the  $2 \times 2$  games, randomization device pro-  
 265 posal (for Maxent is mainly supported in data) is dominate; on the contrary,  
 266 the population proposal is significant less supported.

## 267 5.3. Implication in Experimental Economics

268 Entropy measurement has been shown its useful, e.g., ditinguishing the  
 269 coordination in games [11, 10]. The Maxent methods provides a accuracy way  
 270 to detect distribution. For example, in the experiments of testing a funda-  
 271 mental concept of evolutionary game theory [20], we expect, the deviation of  
 272 the distribution from Nash (in random and independent hypothesis) could be

273 measured out by Maxent method (using the equations in Section 3). Mean-  
274 while, non-trivial white noise (non-gaussian) phenomena has its extremely  
275 interested in theories from multidisciplines [27, 28]. The experimental Max-  
276 ent distributions could be an encouragement for the experiments on behavior  
277 noise.

278 We notice that, Maxent methods are not well known in experimental  
279 economics. Our finding suggests that, referring to Maxent prediction, to test  
280 the entropy and distribution of experimental social outcomes is a practical  
281 way.

#### 282 5.4. *Summary*

283 This report provides firstly experimental evidence of supporting Maxent  
284 in  $2 \times 2$  population games. The report includes also the precision and the  
285 deviation pattern of the experimental social outcomes meet Maxent predic-  
286 tions. We wish our experimental finding could strength the bridges from  
287 game theory to Maxent and to general science.

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